**Honors in Computational Finance** **Semester: VIII**

**Program: Computer Science and Engineering (Data Science)**

**Course: Stochastic Calculus (DJ19DSHN1C4)**

**Pre-requisite:**

1. Calculus I and II
2. Probability and Statistics
3. Linear Algebra
4. Differential Equations

**Objectives:**

1. To familiarize students with various types of random processes and their applications in engineering, finance, and other fields.
2. To explore the concept of Brownian motion as a fundamental stochastic process and its significance in modeling random phenomena.
3. Introduce students to the applications of stochastic calculus in areas such as finance, risk assessment, and engineering optimization.

**Outcomes:** On completion of the course, the learner will be able to:

1. Apply probability theory to analyze random events and situations, and compute probabilities for various scenarios.
2. Describe Brownian motion, understand its properties, and apply it in the context of stochastic processes.
3. Apply numerical methods, including Euler-Maruyama and Monte Carlo simulations, for practical stochastic modeling and analysis.

| **Stochastic Calculus (DJ19DSHN1C4)** | | |
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| **Unit** | **Description** | **Duration** |
| **1** | **Introduction to Probability:** Sample spaces, events, and probability, Conditional probability, Random variables, probability distributions, Joint Distribution of Random Variables, Problems in Random variables and Distributions, Definition of random processes, Stationary and non-stationary processes. | 06 |
| **2** | **Markovian Chain:** Introduction to Discrete time Markov Chain, Introduction to Chapman-Kolmogorov equations, State Transition Diagram and Examples, Introduction to Classification of States and Periodicity, Introduction to Continuous time Markov Chain, Markov Chain Monte Carlo methods. | 10 |
| **3** | **Brownian Motion:** Introduction to Brownian motion, Process derived from Brownian motion, Stochastic differential equation, Properties of Brownian motion, Ito Integrals. | 06 |

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| **4** | **Ito calculus for Brownian motion:**<br>Adapted functions of Brownian motion, filtration (informal), the Ito integral, causal approximation, almost sure convergence, Borel Cantelli lemma, convergence of Ito integral approximation (informal), Ito's lemma, martingales, quadratic variation, Option pricing with fractional Brownian motion. | 10 |
| **5** | **Diffusion processes:**<br>The Ornstein Uhlenbeck process, geometric Brownian motion, backward equation and the generator, forward equation, adjoint, probability dynamics, Ito integral and Ito's lemma for general diffusions (informal), quadratic variation, stochastic differential equations, coupling to Brownian motion, simulating general diffusions, random walk and finite difference approximations, Case studies on Credit risk modeling, High-frequency trading | 10 |
| 6 | **Change of measure:**<br>Probability space and measure (informal), expectation and the probability integral (informal), probability density and likelihood ratio hypothesis testing, the Neyman Pearson lemma, importance sampling, change of measure/drift for Brownian motion, Girsanov theorem for general diffusions, rare event simulation, Applications of Stochastic Calculus to Finance (Risk Neutral Pricing with Geometric Brownian Motion). | 10 |
| | **Total** | **52** |

**Books Recommended:**

*Text books:*


1. Steven Shreve, "Stochastic Calculus for Finance The binomial asset pricing model. I " W. Ross MacDonald School Resource Services Library, 3rd Edition, 2019.
2. Gregory F. Lawler, "Stochastic Calculus: An Introduction with Applications", Gregory F. Lawler, 2023.


*Reference Books:*


1. Sheldon M. Ross, "Introduction to Probability Models", 12th Edition, Academic Press 2022.
2. Samuel N. Cohen, Robert J. Elliott, "Stochastic Calculus and Applications" Springer, 2015.


*Web Links:*

1. NPTEL Course: Stochastic Processes: https://nptel.ac.in/courses/111102098

Prepared by Checked by Head of the Department Principal